Dependence of the BCS $^1S_0$ superfluid pairing gap on nuclear interactions

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Abstract

We study in detail the dependence of the $^1S_0$ superfluid pairing gap on nuclear interactions and on charge-independence breaking at the BCS level. Starting from chiral effective-field theory and conventional nucleon-nucleon (NN) interactions, we use the renormalization group to generate low-momentum interactions $V_{\mathrm{low}\,k}$ with sharp and smooth regulators. The resulting BCS gaps are well constrained by the NN scattering phase shifts, and the cutoff dependence is very weak for sharp or sufficiently narrow smooth regulators with cutoffs $\Lambda > 1.6\,\mathrm{fm}^{-1}$. It is therefore likely that the effect of three-nucleon interactions on $^1S_0$ superfluidity is small at the BCS level. The charge dependence of nuclear interactions has a 10% effect on the pairing gap.

1   Introduction

Superfluidity plays a central role in strongly-interacting many-body systems ranging from nuclei, halo nuclei and neutron stars to cold atoms: The isospin dependence of nuclear pairing gaps shows striking trends over a range of isotopes [1], the β decay of the two-neutron halo in ^{11}Li is suppressed due to pairing [2] similar to neutrino emission in neutron star cooling [3], and resonant Fermi gases exhibit vortices [4] and superfluid characteristics in thermodynamic [5] and spectroscopic properties [6].

For relative momenta $k \lesssim 2\,\mathrm{fm}^{-1}$, nucleon-nucleon (NN) interactions are well constrained by the existing scattering data [7]. The model dependence for

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larger momenta shows up prominently, for instance, in the $^3P_2$ superfluid pairing gaps for Fermi momenta $k_F > 2\,\mathrm{fm}^{-1}$ [8]. However, some uncertainty remains concerning a possible dependence of the $^1S_0$ pairing gap on the input NN interaction in low-density neutron matter ($k_F < 1.6\,\mathrm{fm}^{-1}$). In this letter, we clarify this point and explore the dependence of $^1S_0$ superfluidity on nuclear interactions at the BCS level in detail. We find that the BCS gap is well constrained by the NN phase shifts. Therefore, any uncertainties are due to polarization (induced interaction), dispersion and three-nucleon (3N) interaction effects.

In addition to chiral effective-field theory (EFT) and conventional NN interactions, we use the renormalization group (RG) to evolve nuclear interactions to a lower resolution scale. The resulting class of low-momentum interactions $V_{\mathrm{low}\,k}$ [7,9,10], which is defined by a regulator with a variable cutoff Λ, reproduces the NN scattering phase shifts for momenta below Λ. We find that the cutoff dependence of the $^1S_0$ BCS gap is very weak for sharp or sufficiently narrow smooth regulators with $\Lambda > 1.6\,\mathrm{fm}^{-1}$. A comparison with the cutoff dependence found in $A = 3, 4$ nuclei ("Tjon-line") [11] and in nuclear matter [12], when 3N interactions are neglected, suggests that 3N interaction effects on $^1S_0$ superfluidity are small at the BCS level.

Contact or separable pairing interactions can be implemented directly in current density-functional calculations. For low-momentum interactions, the weak-coupling approximation with a density-dependent contact interaction is reliable (see Ref. [13]), and a separable approximation is efficient [14]. Therefore, low-momentum interactions offer the possibility for a consistent treatment of the particle-hole and pairing channels in density-functional theory. Moreover, it is straightforward to adapt the RG to microscopically derive the renormalized pairing interaction introduced in the optimal regularization scheme of Bulgac [15].

This letter is organized as follows. In Sect. 2, we present the formalism for $^1S_0$ superfluidity at the BCS level and our results for the dependence of the pairing gap on nuclear interactions and for the effects of charge-independence breaking. We conclude in Sect. 3. We emphasize that our work should be considered as a theoretical benchmark and not as a prediction of the superfluid pairing gap, since we do not include contributions beyond the BCS level.

2   Formalism and results at the BCS level

In the BCS approximation, the $^1S_0$ superfluid gap $\Delta(k)$ is obtained by solving the gap equation with a free spectrum, $\varepsilon(p) = p^2/2$ (in units $c = \hbar = m = 1$,

with m the nucleon mass),

$$\Delta(k) = -\frac{1}{\pi} \int dp\, p^2 \, \frac{V(k,p)\,\Delta(p)}{\sqrt{\xi^2(p) + \Delta^2(p)}} \,, \tag{1}$$

where $V(k,k')$ is the free NN interaction, $\xi(p) \equiv \varepsilon(p) - \mu$, and for a free spectrum the chemical potential is given by $\mu = k_{\mathrm{F}}^2/2$. The Lippmann-Schwinger equation for the scattering T matrix in the same channel reads

$$T(k,k';E) = V(k,k') + \frac{2}{\pi} \int dp\, p^2 \, \frac{V(k,p)\,T(p,k';E)}{E - p^2} \,. \tag{2}$$

Here and in the following, principal value integrals are implied. The homogeneous gap equation can be understood as an equation for the residue of the pole of the T matrix, $T(k,k';E) \to \Delta(k)\Delta(k')/[E - 2\mu]$, for $E \to 2\mu$, when the two-nucleon propagator in Eq. (2) is replaced by the corresponding self-consistent Nambu-Gorkov propagator $2\mu - p^2 \to -2\sqrt{\xi^2(p) + \Delta^2(p)}$. This includes the propagation of back-to-back particle-particle, $(1 - n_p)(1 - n_p)$, and hole-hole modes, $-n_p\, n_p$, where n_p denotes the Fermi-Dirac distribution.

For conventional large-cutoff and chiral EFT potentials, $V(k,k')$ includes regulating functions that render the integral convergent. These are of exponential form $\exp[-(k^2/\Lambda^2)^3]$ with $\Lambda = 450 - 600\,\mathrm{MeV}$ in the current chiral EFT interactions at N^{3}LO [16,17], and phenomenological functions that imply large (few GeV) cutoffs in conventional NN potential models.

For the RG evolution to low-momentum interactions $V_{\mathrm{low}\,k}$, we define a reduced interaction $v(k,k')$ with $V_{\mathrm{low}\,k}(k,k') = f(k)\,v(k,k')\,f(k')$, where $f(k)$ denotes a sharp or smooth regulator. Starting from an NN interaction with a large cutoff $v(k,k') = V_{\mathrm{NN}}(k,k')$, we use the RG equation [10],

$$\frac{d}{d\Lambda}\, v(k',k) = \frac{1}{\pi} \int_0^\infty p^2 dp \left[\frac{v(k',p)\frac{d}{d\Lambda}[f^2(p)]\,t(p,k;p^2)}{p^2 - k^2} \right.$$
$$\left. + \frac{t(k',p;p^2)\frac{d}{d\Lambda}[f^2(p)]\,v(p,k)}{p^2 - k'^2} \right], \tag{3}$$

to generate low-momentum interactions with a variable cutoff Λ (both v and f depend explicitly on Λ). Here, the reduced t matrix $t(k,k';E)$ is defined by $T_{\mathrm{low}\,k}(k,k';E) = f(k)\,t(k,k';E)\,f(k)$, where $T_{\mathrm{low}\,k}$ is the solution to Eq. (2) with $V = V_{\mathrm{low}\,k}$. In this letter, we consider a sharp cutoff $f(k) = \theta(\Lambda - k)$ and smooth regulators of the exponential form $f(k) = \exp[-(k^2/\Lambda^2)^n]$, where n is a parameter that controls the smoothness. The resulting $V_{\mathrm{low}\,k}$ is hermitian and preserves the low-momentum fully-on-shell T_{NN} matrix, up to factors of the regulator function $T_{\mathrm{low}\,k}(k,k;k^2) = f^2(k)\,T_{\mathrm{NN}}(k,k;k^2)$, as well as the deuteron binding energy [10].

The RG equation, Eq. (3), is equivalent to a generalization of the Lee-Suzuki method [18,19] and a subsequent Okubo hermitization [20] to smooth cutoffs [10,21]. In the following, we will use the projection operator formalism to construct low-momentum interactions with a sharp cutoff and solve the RG equation for our results obtained with a smooth regulator. We note that the RG equation cannot be solved directly in the neutron-neutron $^1S_0$ channel for most of the conventional NN interactions (except for the Nijmegen II potential), due to spurious resonances at high ($\sim$ GeV) momenta. The freedom in the choice of the regulator $f(k)$ implies a scheme dependence of the gap $\Delta(k) \sim f(k)$ at large momenta $k \gg k_F$ (see Eq. (1)). We will restrict our results to the gap on the Fermi surface $\Delta \equiv \Delta(k_F)$, where the momenta are on-shell and the gap is scheme independent.

In the leading-order pionless EFT with sharp-cutoff regularization, one has $V(k, k') = \theta(\Lambda - k)\,\theta(\Lambda - k')/[1/a_s - 2\Lambda/\pi]$ (with scattering length a_s). The resulting gap is cutoff independent for $\Delta \ll \mu$ and large cutoffs, which follows from the gap equation with $\Delta(k) = \theta(\Lambda - k)\,\Delta$,

$$
\frac{1}{a_s} - \frac{2}{\pi}\Lambda = -\frac{1}{\pi}\int_0^\Lambda dp\, \frac{p^2}{\sqrt{\xi^2(p) + \Delta^2}}\,. \tag{4}
$$

For $\Delta \ll \mu$ and large Λ, the integral is given by $2\left[-2k_F + \Lambda + k_F \ln(8\mu/\Delta)\right]$. The UV divergence cancels against the cutoff dependence of the interaction in Eq. (4). This leads to the standard result $\Delta = 8\mu/e^2 \exp[\pi/(2k_F a_s)]$ [22].

For the solution of the gap equation, we follow the method of Khodel *et al.* [23]: We first decompose the interaction into a separable and a non-separable part

$$
V(k, k') = V(k_F, k_F)\,\phi(k)\,\phi(k') + W(k, k')\,, \tag{5}
$$

where $\phi(k) \equiv V(k, k_F)/V(k_F, k_F)$ and $W(k, k')$ is a suitably chosen function that vanishes when at least one argument is on the Fermi surface ($k = k_F$). Then the gap equation, Eq. (1), can be replaced by an equivalent system of two equations,

$$
\phi(k) = \chi(k) + \frac{1}{\pi}\int dp\, p^2\, \frac{W(k, p)\,\chi(p)}{\sqrt{\xi^2(p) + \Delta^2\,\chi^2(p)}}\,, \tag{6}
$$

$$
0 = 1 + V(k_F, k_F)\frac{1}{\pi}\int dp\, p^2\, \frac{\phi(p)\,\chi(p)}{\sqrt{\xi^2(p) + \Delta^2\,\chi^2(p)}}\,, \tag{7}
$$

where $\Delta(k) \equiv \Delta\,\chi(k)$ with $\chi(k_F) = 1$. This system has the advantage that the integrand in Eq. (6) vanishes on the Fermi surface, and consequently the function $\chi(k)$ is only weakly sensitive to changes of $\Delta(p)$ in the denominator. Therefore, to a good approximation, Eq. (6) can be linearized. In the
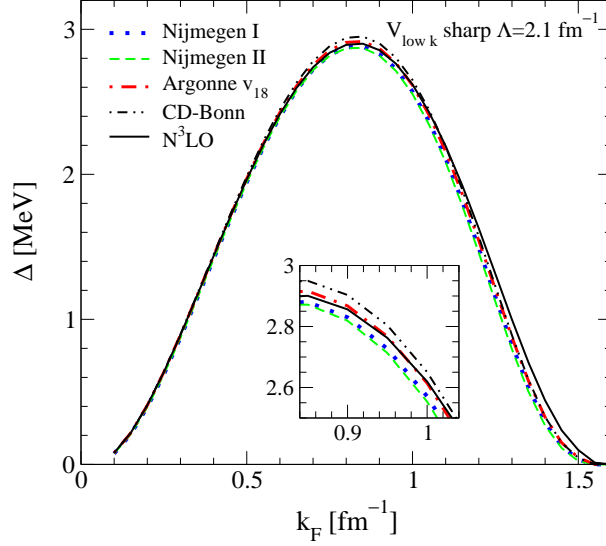
Fig. 1. The neutron-neutron $^1S_0$ superfluid pairing gap on the Fermi surface $\Delta \equiv \Delta(k_F)$ versus Fermi momentum k_F for low-momentum interactions $V_{\text{low }k}$ with a sharp cutoff $\Lambda = 2.1\,\text{fm}^{-1}$. $V_{\text{low }k}$ is derived from various charge-dependent NN interactions [16,24,25,26]. We have verified that the results are cutoff independent from $\Lambda = 1.6\,\text{fm}^{-1}$ to $\Lambda = 2.5\,\text{fm}^{-1}$. The inset magnifies the small dependence on nuclear interactions near the maximum.

first iteration, we solve Eq. (6) by inversion using a sufficiently small constant Ansatz for $\Delta(p)$ in the denominator. Next, we solve Eq. (7) for Δ using Newton's method with the solution for $\chi(k)$ of the previous step. The iteration of this procedure (where $\Delta(p)$ in the denominator of Eq. (6) is updated at each step) leads to a rapidly converging solution for the BCS gaps. We follow this method for the smooth-cutoff $V_{\text{low }k}$. For the sharp-cutoff case, we have found that a solution of the gap equation, Eq. (1), by iteration with an initial guess $\Delta(k) \sim \sqrt{|k_F\, V_{\text{low }k}(k,k)|}$ also converges rapidly. This Ansatz is motivated by the observation that a separable approximation is meaningful for low-momentum interactions [14].

Our results for the density dependence of the neutron-neutron $^1S_0$ superfluid gap Δ are shown in Fig. 1. The low-momentum interactions $V_{\text{low }k}$ are derived from various charge-dependent NN potentials [16,24,25,26] using a sharp cutoff $\Lambda = 2.1\,\text{fm}^{-1}$. We find that the BCS gap is almost independent of the NN interaction. Consequently, we conclude that the $^1S_0$ gap is strongly constrained by the NN scattering phase shifts. This has been noted previously (see for example Ref. [27]), but without considering charge dependences. Moreover, these are the first results for chiral interactions at N^{3}LO. We use the N^{3}LO chiral potential of Ref. [16], since it is the chiral interaction that leads to the most accuracte reproduction of the phase shifts.

The maximal gap at the BCS level is $\Delta \approx 2.9-3.0\,\text{MeV}$ for $k_F \approx 0.8-0.9\,\text{fm}^{-1}$. The small deviation of the N^{3}LO gap from the band at higher densities in
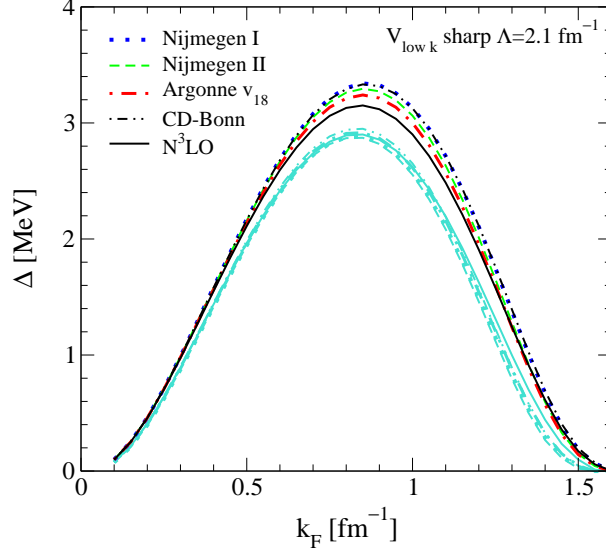
Fig. 2. The charge dependence of the $^1S_0$ superfluid pairing gap Δ versus k_F. The lines indicated in the legend are the neutron-proton gaps, whereas the grey lines show the neutron-neutron gaps from Fig. 1. For further details, see the caption of Fig. 1. We have also verified that the neutron-proton gaps are cutoff independent over the same sharp-cutoff range.

Fig. 1 is consistent with the slightly more attractive $^1S_0$ phase shifts at the corresponding energies (compare, for example, the phase shifts of the CD-Bonn [26] and N^{3}LO potentials). We find that the gaps are cutoff independent over the range considered here, $\Lambda = 1.6\,\text{fm}^{-1}$ to $\Lambda = 2.5\,\text{fm}^{-1}$. This result is consistent with the findings of Kaiser *et al.* that the cutoff dependence is substantially reduced for chiral EFT interactions when going from NLO to N^{2}LO [28], since the latter leads to a better description of the NN scattering phase shifts. In addition, the BCS gaps for the "bare" interactions are within 2% of the $V_{\text{low}\,k}$ results shown in Fig. 1 for $k_F \lesssim 1.0\,\text{fm}^{-1}$, and the difference is compatible with the spread in the $V_{\text{low}\,k}$ result over all densities. (This also holds for Fig. 2.) For completeness, we mention that Sedrakian *et al.* [29] have solved the BCS gap equation for one low-momentum interaction ($V_{\text{low}\,k}$ derived from Nijmegen 93 [24] with $\Lambda = 2.5\,\text{fm}^{-1}$), but they did not explore the cutoff dependence.

Isospin symmetry breaking leads to small charge dependences in nuclear interactions. As a result, the $^1S_0$ neutron-proton scattering length $a_{\text{np}} = -23.768 \pm 0.006\,\text{fm}$ [30] is more attractive than the neutron-neutron scattering length $a_{\text{nn}} = -18.5 \pm 0.3\,\text{fm}$ [31] in the same channel. This effect is dominantly due to the charge dependence of the one pion-exchange interaction V_π. The central part of V_π in the neutron-proton charge-exchange channel is of the form $-m_{\pi\pm}^2/(q'^2 + m_{\pi\pm}^2)$, where q' is the (exchange) momentum transfer. Since the charged pion is heavier than the neutral one, $m_{\pi\pm} = 139.57\,\text{MeV}$ and $m_{\pi^0} = 134.98\,\text{MeV}$, the resulting neutron-proton interaction is more attrac-
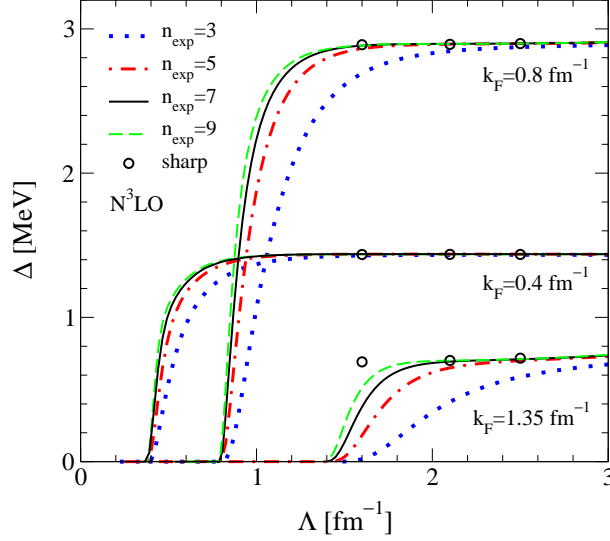
Fig. 3. The neutron-neutron $^1S_0$ superfluid pairing gap Δ as a function of the cutoff Λ for three densities and different smooth exponential regulators, as well as for a sharp cutoff. The low-momentum interactions are derived from the N^{3}LO chiral potential of Ref. [16].
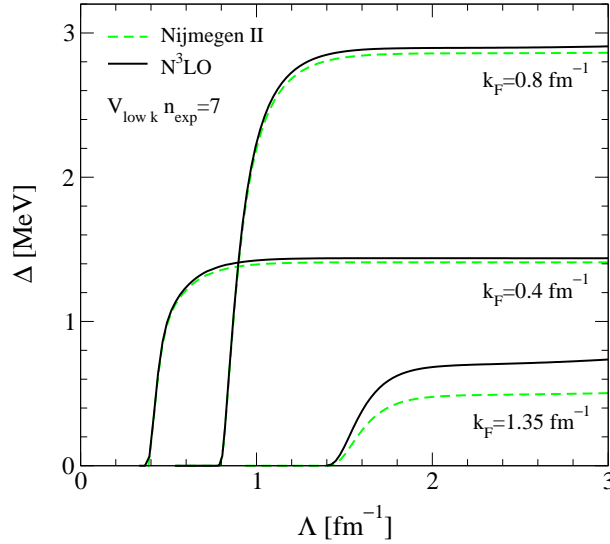

Fig. 4. The neutron-neutron $^1S_0$ superfluid pairing gap Δ as a function of the cutoff Λ for three densities. The low-momentum interactions are derived from the N^{3}LO [16] and the Nijmegen II [24] potential with exponential regulator $n = 7$.

tive. In Fig. 2, we show the charge dependence of the $^1S_0$ superfluid pairing gap versus Fermi momentum. The neutron-proton gaps are ≈ 0.3 MeV larger at maximum with a slight shift to higher densities. The 10% effect on the pairing gaps clearly reflects the charge dependence of nuclear interactions.

Next, we study the dependence of the neutron-neutron $^1S_0$ superfluid pairing gap as a function of the cutoff starting from the N^{3}LO chiral interaction. Our results for three representative densities and different smooth exponential

regulators $f(k) = \exp[-(k^2/\Lambda^2)^n]$, as well as for a sharp cutoff, are shown in Fig. 3. As long as the cutoff is large compared to the dominant momentum components in the Cooper bound state, the gap depends very weakly on the cutoff. Below this scale, which depends on the density and the smoothness of the regulator, the strength of the bound state decreases, since some of the momentum modes that build up the Cooper pairs are integrated out. From Fig. 3, we observe that the cutoff dependence is very weak for sharp or sufficiently narrow smooth regulators with $\Lambda > 1.6\,\mathrm{fm}^{-1}$. It can be seen that $n = 3$ is too smooth, but that $n > 5$ is sufficient. For lower densities, even lower cutoffs with $\Lambda > 1.2 k_{\mathrm{F}}$ are possible.

The $\mathrm{N}^3\mathrm{LO}$ chiral interaction has a cutoff $\Lambda = 2.5\,\mathrm{fm}^{-1}$ (or $500\,\mathrm{MeV}$) [16] and one may suspect that the cutoff dependence could be larger for conventional NN potentials. In Fig. 4, we show that this is not the case by comparing the gaps from Fig. 3 to results obtained with the Nijmegen II potential [24], which has a large ($\sim \mathrm{GeV}$) cutoff. The resulting cutoff dependences are similar and in particular very weak for sharp or sufficiently narrow smooth regulators with $\Lambda > 1.6\,\mathrm{fm}^{-1}$. This shows that the $^1\mathrm{S}_0$ superfluid pairing gap probes low-momentum physics.

3   Conclusions

In this Letter, we have presented a systematic study of the interaction dependence of the BCS $^1\mathrm{S}_0$ superfluid pairing gap. We have shown that this gap is practically independent of the choice of the NN interaction, and therefore well constrained by the NN phase shifts. Furthermore, we have found only a very weak dependence on the cutoff for low-momentum interactions $V_{\mathrm{low}\,k}$ with sharp or sufficiently narrow smooth regulators for $\Lambda > 1.6\,\mathrm{fm}^{-1}$. For low densities, it is possible to lower the cutoff further to $\Lambda > 1.2 k_{\mathrm{F}}$. We also find that the pairing gap clearly reflects the charge dependence of NN interactions. Neutron-neutron and neutron-proton $^1\mathrm{S}_0$ are not carefully distinguished in previous work. We conclude that the uncertainties in $^1\mathrm{S}_0$ superfluidity are due to an approximate treatment of induced interactions and dispersion effects, which go beyond the BCS level, as well as due to 3N interactions.

The weak cutoff dependence indicates that, in the $^1\mathrm{S}_0$ channel, the contribution of 3N interactions is small at the BCS level. We note that the 2π-exchange part ("c_i-terms") of the corresponding low-momentum 3N interactions [11] is cutoff independent up to the regulator functions. The latter lead to a cutoff dependence as the density increases (see, for example, the Hartree-Fock results in Ref. [12]). We also emphasize that 3N interactions will contribute differently to $^1\mathrm{S}_0$ superfluidity in pure neutron matter compared to symmetric nuclear matter. Additional insights will come from an investigation of the

cutoff dependence of $^3S_1$–$^3D_1$ superfluidity, where 3N interactions may play an important role. Work in this direction is in progress [32].

Low-momentum interactions, via weak-coupling or separable approximations, can be implemented directly in current density-functional calculations. Furthermore, it is straightforward to adapt the RG used here to a microscopic derivation of the optimal pairing interaction of Ref. [15].

Finally, we emphasize that our results are obtained at the BCS level and do not include polarization and self-energy effects. Therefore, our work should be considered as a theoretical benchmark and not as a prediction of the superfluid pairing gap. Recently, polarization effects on the pairing gap have been studied in an RG approach to the many-body problem, starting from low-momentum NN interactions [13]. Is is found that polarization effects lead to a suppression to a maximal gap $\Delta \approx 0.8\,\mathrm{MeV}$, in qualitative agreement with the earlier work of Wambach *et al.* [33]. The RG approach is nonperturbative and includes long-range particle-hole induced interactions and the dominant self-energy effects. Tensor induced interactions are not included in Ref. [13] because their effects on $^1S_0$ superfluidity are expected to be small at very low densities, and because the strength of the tensor force is coupled to the corresponding 3N interaction [10,11]. In addition, there are recent finite-particle-number, fixed-node AFD Monte Carlo calculations [34], which in principle include olarization effects, but result in a maximal gap $\Delta \approx 2.5\,\mathrm{MeV}$. However, in these calculations larger particle numbers may be required to capture long-range polarization effects.

Acknowledgements

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